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**Bayesian Final Presentation**

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“An Overview of Robust Bayesian Analysis” gives the motivation and techniques for Robust Bayesian Analysis (RBA). RBA is the study of the sensitivity of Bayesian answers to uncertain inputs (e.g. model and prior). RBA is important because it gives a response to the non-Bayesian critique that arbitrary discriminations of uncertain inputs cannot be made. RBA agrees with the critique yet still provides a realistic foundational system that allows subjective Bayesian analysis to proceed. Practically, RBA is important because it tells you the degree of elicitation that is necessary, and what to elicit. For example, RBA can tell you that further elicitation is unnecessary if the corresponding range of Bayesian answers is appropriate, or that only a couple parameters out of many influence the result and require elicitation. RBA also alleviates fear of the subjective prior as it reflects elicitor’s uncertainty. In addition, it has been shown that when Bayes procedures are robust they have “excellent frequentist properties.” Section 1 gives two examples that show RBA’s ability to provide a better interpretation to classical p-values, and fix problematic Neyman-Pearson tests.

Section 2 discusses the choice of robust statistical model/prior distributions. An example is given showing that prior/model choice is often arbitrary and standard choices (e.g. exponential family model and conjugate priors) are typically non-robust. Flat tailed priors are more robust than standard choices. An example is given showing that replacing a normal model and prior with a t-distribution and Cauchy distribution is more robust because the new distributions have thicker tails. A similar idea can apply to multivariate models and priors. Non-informative priors also yield robust answers since they are designed to have minimal influence on the answer. Likewise, partially informative priors are often robust and can be setup in two ways: parameters of interest are elicited while nuisance parameters are ignored, or prior/model set up with certain features and then prior/model is chosen which maximizes entropy (subject to constraints). Finally, Bayesian non-parametrics are an approach to automatic robustness as they automatically adapt to the true model (e.g. Dirichlet process prior or Gaussian process priors with smoothing splines). Caution is advised with non-parametric prior/models because as the sample size goes to infinity the estimates of quantities doesn’t necessarily convert to true values (example 4 in paper).

Section 3 discusses diagnostics. The goal of diagnostics is to detect if robustness problems exists and, if so, where the difficulty lies. Diagnostics are typically based on utilization of . The paper does not go into depth around diagnostics but provides references to outside papers. Influence and sensitivity is then discussed, which is when features of model and priors have a large effect on the Bayesian answer. A current approach to sensitivity is to consider functional derivatives of the Bayes operator with respect to . These derivatives, evaluated at a base prior and in “direction” g, indicate how sensitive is to local changes in . References are provided to other papers for more information.

Section 4 discusses how to calculate global robustness, which assumes , , and and are classes of densities. Global robustness is reported as a range of values, , where and . If the range is small enough, you can stop. If not, further elicitation of model / prior is required. and is often defined as a parametric density class. Parametric classes are attractive because they are easy to compute and communicate, though may fail to capture realistic possible deviations from the base model or prior. Most RBA is focused on holding fixed and setting the class of because the prior is less well known than the model (addressing prior uncertainty), and it reflects variety of prior opinions. The paper suggest four factors when choosing a class of prior: (1) easy to elicit and interpret, (2) computationally as easy as possible, (3) size of class should reflect prior uncertainty or else erroneous conclusions are possible, and (4) should be extendable to higher dimensions. An example is given showing the erroneous conclusions when is too large or too small. Section 4 then discusses the following common classes for priors.

* Classes of Given Shape or Smoothness: . Often used in conjunction with large class that needs to be made smaller.
* Moment Class: . Difficult to elicit and typically used to constrain larger class.
* Contamination Class: where is base prior, is perceived possible error in base prior, and Q is the allowed class of contamination. Easy to elicit and compute. Class can become too big if Q or is appreciable.
* Density Ratio Class: . Hard to elicit choice of L and U but simplest to handle computationally.
* Quantile Class: . Natural to elicit.
* Mixture Class: . Mixtures play prominent role because computationally simple, flexible, not too big, and can be easily used in refinement step of elicitation.
* Marginal and Independence Class: . Typically enormous and often you specify independence and .
* Other classes: distribution band, neighborhood, belief function, Choquet Capacities, and near ignorance class.

Section 4 then discusses the global robustness approach applied to hypothesis testing, which can provide lower bounds on the Bayes factor for different p-values. An example shows the surprisingly large discrepancy between p-values and Bayes factors. Global robustness can also be applied to likelihoods, but with difficulties. Overall, the limitation of global robustness is that it ignores a very important quantity, , which can lead to erroneous results if a full Bayesian analysis is not done. Section 4 concludes by discussing the optimization of global robustness, which is done by specifying a credible set C, for , and requiring that the posterior probability of C satisfy

Section 5 discusses computational tricks such as linearization to find , and reweighting to quickly go between priors. The future technical goal of RBA is to create an interactive Bayesian program that can proceed with partial information and provide implied range of Bayesian answers, suggesting what additional elicitations are desirable.

Section 6 concludes by saying RBA can be applied when subjective Bayes is feasible, and is necessary when objective Bayesian methods do not exist. RBA is also intuitively applicable to clinical trials and group decision-making since there are a variety of prior opinions in both cases.